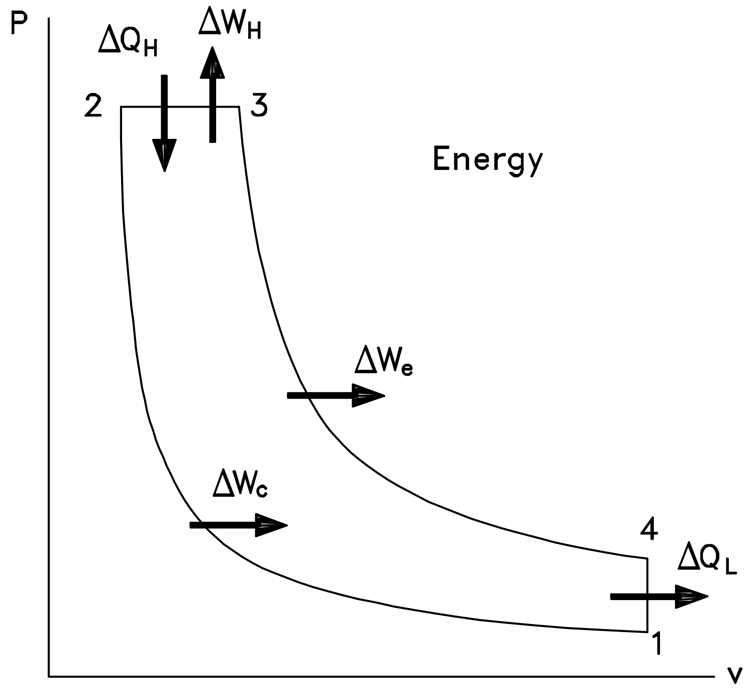
Problem: 6-5

Given: At the start of the compression process in a Diesel-cycle engine the air is at 530 R, 14.7 psia. The compression ratio is 16. The peak cycle temperature is equal to the 4000 R source temperature. The adiabatic compression and expansion processes are reversible.

Find: i) the energy transfer per unit mass for each process in the cycle

Assume: Even though high temperatures are involved, assume that specific heats are constants at their low-temperture values.



Solution:
The analysis begins with the evaluation of properties.

State	T (K)	P (MPa)	v (m^3/kg)
1	530	14.7	
2			
3	4000		
4			

The specific volume at state (1) may be computed from the ideal-gas equation as:

$$
\begin{aligned}
v_1 &= R\,T_1/P_1 \\
&= \frac{53.35\ ft - lbf/(lbm - R)\ \times 530\ R}{14.7\ lbf/in^2\ \times\ 144\ in^2/ft^2} \\
&= 13.36\ ft^3/lbm
\end{aligned}
$$

An entropy balance for the reversible, adiabatic compression process gives $s_1 = s_2$. The compression ratio gives $v_1/v_2 = 16.0$ The value of T_2 may be found by integrating $T\ ds = du + P\ dv$ for the isentropic process from state (1) to state (2) to obtain:

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Upon multiplying by T_1 and substituting numerical values

$$T_2 = 530 \times 16.0^{(1.4-1)} = 1606.7\ R$$

The pressure after compression may be found from the ideal gas equation as

$$P_2 = m_2\ R\ T_2/V_2$$

Since $m_2 = m_1 = (P_1V_1)/(R\,T_1)$ we may write

$$
\begin{aligned}
P_2 &= [(P_1V_1)/(R\,T_1)][R\,T_2/V_2] \\
&= P_1(V_1/V_2)(T_2/T_1) \\
&= 14.7 \times 16.0 \times (1606.7/530) \\
&= 713.0\ psia
\end{aligned}
$$

The specific volume at state (2) may be computed from the compression ratio as

$$v_2 = v_1/(v_1/v_2) = 13.36/16.0 = 0.835\ ft^3/lbm$$

Since T_3 is given as 4000 R and $P_3 = P_2$, v_3 may be found as

$$
\begin{aligned}
v_3 &= R\,T_3/P_3 \\
&= \frac{53.35\ ft - lbf/(lbm - R)\ \times 4000\ R}{713.0\ lbf/in^2\ \times\ 144\ in^2/ft^2} \\
&= 2.078\ ft^3/lbm
\end{aligned}
$$

State (4) is fixed by $s_4 = s_3$ and $v_4 = v_1$. The value of T_4 can be found by integrating $T\ ds = du + P\ dV$ for the isentropic process from state (3) to state (4) to obtain

$$\frac{T_4}{T_3} = \left(\frac{v_4}{v_3}\right)^{1-k}$$

Since $v_4 = v_1$ we may substitute values for v_1 and v_3 to obtain

$$T_4 = 4000 \times (13.36/2.078)^{(1-1.4)} = 1900.2\ R$$

The pressure at state (4) is given by

$$P_4 = R\ T_4/v_4$$

Since $v_4 = v_1 = R\ T_1/P_1$ we may write

$$P_4 = (T_4/T_1)P_1 = (1900/530) \times 14.7 = 52.7\ psia$$

The table of property values may now be completed.

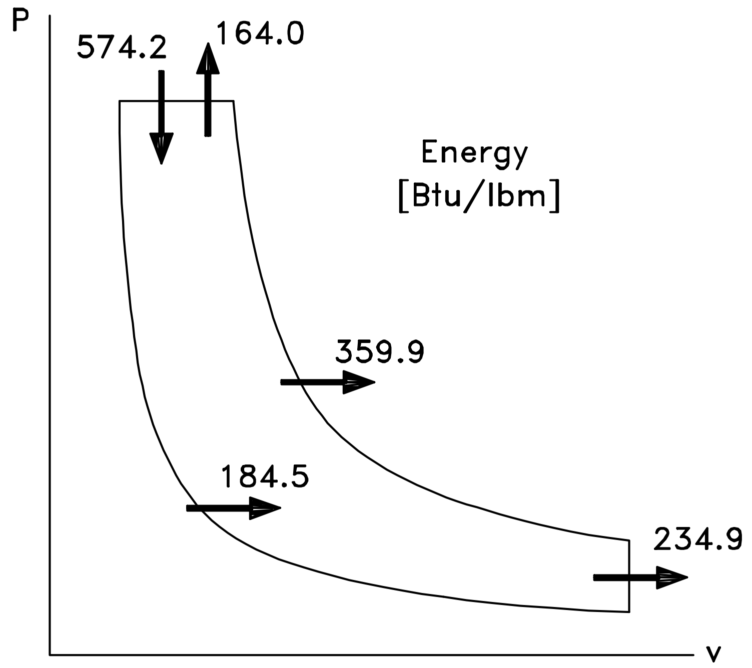
State	$\boldsymbol{T}$ $\boldsymbol{(K)}$	$\boldsymbol{P}$ $\boldsymbol{(MPa)}$	$\boldsymbol{v}$ $\boldsymbol{(m^3/kg)}$
1	530	14.7	13.36
2	1606.7	713.0	0.835
3	4000	713.0	2.078
4	1900.2	52.7	13.36

For constant specific heats, the energy transfers per unit mass may be found as:

$$\begin{aligned}
\Delta W_c/m = &\ c_v(T_2 - T_1) = 0.1714\ (1606.7 - 530) &= 184.5\ Btu/lbm \\
\Delta Q_H/m = &\ c_p(T_3 - T_2) = 0.2399\ (4000 - 1606.7) &= 574.2\ Btu/lbm \\
\Delta W_e/m = &\ c_v(T_3 - T_4) = 0.1714\ (4000 - 1900.2) &= 359.9\ Btu/lbm \\
\Delta Q_L/m = &\ c_v(T_4 - T_1) = 0.1714\ (1900.2 - 530) &= 234.9\ Btu/lbm
\end{aligned}$$

For the heating process

$$\begin{aligned}
\Delta W_H/m &= P_o(v_3 - v_2) \\
&= \frac{713.0\ lbf/in^2 \times (2.078 - 0.835)\ ft^3/lbm \times 144\ in^2/ft^2}{778.2\ Btu/ft - lbf} \\
&= 164.0\ Btu/lbm
\end{aligned}$$


P
574.2
164.0
Energy
[Btu/lbm]
359.9
184.5
234.9
v