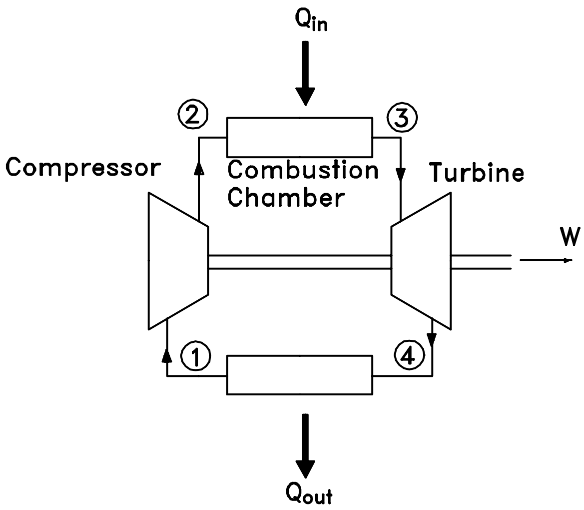
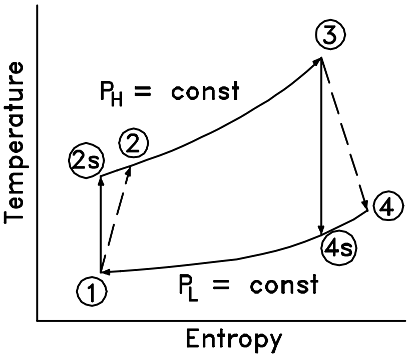
Problem: 7-4

Given: A simple gas turbine engine operates with air as the working fluid, a pressure ratio of 18:1 and a maximum temperature of 700 °C. The air enters the compressor at 100 kPa, 20°C. The compression and expansion isentropic efficiencies are 0.87 and 0.94, respectively. The dead state has a $T_0 = 20$ °C and a $P_0 = 100\ kPa$.

Find: The specific entropy generation and change in availability for each process in the cycle.

Assume: The temperature of the energy source reservoir is the same as the maximum cycle temperature and the temperature of the energy-sink reservoir is the same as the minimum cycle temperature
Specific heat are considered to be constant.





The various cycle states will be numbered as in the above figure. Beginning at the compressor inlet and working around the cycle to the turbine outlet.

$$P_2 = P_3 = r_p P_1 = 1800\ kPa$$

$$T_{2s} = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = 669\ K$$

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} = 725\ K$$

$$
\begin{aligned}
T_{4s} &= T_3 \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = 426\ K \\
T_4 &= T_3 - \eta_t (T_3 - T_{4s}) = 459\ K
\end{aligned}
$$

The enthalpy, entropy and flow exergy of the air, presuming constant specific heats are

$$
\begin{aligned}
h &= c_p (T - T_0) \\
s &= c_p \ln\left(\frac{T}{T_0}\right) - R \ln\left(\frac{P}{P_0}\right) \\
\psi &= (h - h_0) - T_0 (s - s_0)
\end{aligned}
$$

The various states around the cycle as summarized as follows

State	$\boldsymbol{P\ (kPa)}$	$\boldsymbol{T\ (K)}$	$\boldsymbol{h\ (kJ/kg)}$	$\boldsymbol{s\ (kJ/kg \cdot K)}$	$\boldsymbol{\psi\ (kJ/kg)}$
1	100	293	0	0	0
2	1800	725	433.5	0.07967	410.2
3	1800	973	682.4	0.3749	572.6
4	100	459	166.6	0.4504	34.6

The specific entropy generation for this process is

$$
\frac{\mathcal{P}_{s_{12}}}{m} = \frac{1}{T_0} \left[\frac{Q_{12}}{m} \left(1 - \frac{T_0}{T_j}\right) - \frac{W_{12}}{m} + (h_1 - T_0 s_1) - (h_2 - T_0 s_2) \right]
$$

where T_j is the temperature of any interacting thermal energy reservoir.
For compression $1 - 2$

$$
\begin{aligned}
\frac{W_{12}}{m} &= h_1 - h_2 = -433.5\ kJ/kg \\
\frac{\mathcal{P}_{s_{12}}}{m} &= \frac{1}{293} [433.5 + (0 - 433.5) - 293(0 - 0.07967)] \\
&= 0.07967\ kJ/kg \cdot K \\
\psi_2 - \psi_1 &= 410.2\ kJ/kg
\end{aligned}
$$

For the heat addition process $2 - 3$, presuming that the heat supplying energy reservoir is at $973\ K$

$$\begin{aligned}
\frac{Q_{23}}{m} &= h_3 - h_2 = 248.9\ kJ/kg \\
\frac{\mathcal{P}_{s_{23}}}{m} &= \frac{1}{293}[248.9\left(1 - \frac{293}{973}\right) + (433.5 - 682.4) - 293(0.07967 - 0.3749)] \\
&= 0.03942\ kJ/kg \cdot K \\
\psi_2 - \psi_1 &= 162.4\ kJ/kg
\end{aligned}$$

Across the turbine $3 - 4$

$$\begin{aligned}
\frac{W_{34}}{m} &= h_3 - h_4 = 515.8\ kJ/kg \\
\frac{\mathcal{P}_{s_{34}}}{m} &= \frac{1}{293}[-515.8 + (682.4 - 166.6) - 293(0.3749 - 0.4504)] \\
&= 0.07550\ kJ/kg \cdot K \\
\psi_4 - \psi_3 &= -522.8\ kJ/kg
\end{aligned}$$

For the heat rejection process, $4 - 1$, taking the heat receiving energy reservoir at 293 K

$$\begin{aligned}
\frac{Q_{41}}{m} &= h_1 - h_4 = -166.6\ kJ/kg \\
\frac{\mathcal{P}_{s_{41}}}{m} &= \frac{1}{293}[166.6 - 0) - 293(0.4504 - 0)] \\
&= 0.1162\ kJ/kg \cdot K \\
\psi_1 - \psi_4 &= -34.0\ kJ/kg
\end{aligned}$$

The exergy accounting is as follows where the work potentials were calculated

$$\frac{W_{ptl}}{m} = \Delta\psi + \frac{Q}{m}\left(1 - \frac{T_0}{T}\right)$$

Component	$\boldsymbol{\Delta\psi}$ **kJ/kg**	$\boldsymbol{w_{ptl}}$ **kJ/kg**	$\boldsymbol{w_{act}}$ **kJ/kg**	$\boldsymbol{w_{lost}}$ **kJ/kg**	$\mathcal{P}_s$ **$kJ/kg \cdot K$**
Compressor	410.2	-410.2	-433.5	23.3	0.07967
Combustor	162.4	11.5	0	11.5	0.03942
Turbine	-522.8	522.8	515.8	7.1	0.07550
Atmospheric	-34.0	34.0	0	34.0	0.1162
Total		158.1	82.3	75.9	

The compression could have been done with 410.2 kJ/kg of work, although it actually took 433.5 kJ/kg for this process. Additional work was required to overcome the process irreversibilities as embodied in the compressor efficiency. The net effect of this ineffective use of work was to generate 0.07967 $kJ/kg \cdot K$ of entropy in the compression process. The turbine also does not produce as much work as exergy analysis indicates it can. In the turbine, the impact of irreversibilities is not quite as extensive as in the compressor; this is seen by comparing the specific entropy generations and lost work potentials of the two components.

Adding heat to the constant pressure process from an isothermal energy reservoir is ineffective, as shown by the change in the air's exergy during this process. This should have been expected, because the heat transferred from the reservoir to the air must be transferred across a large temperature difference at the beginning of the process. A similar irreversibility occurs in the heat-rejecting process.