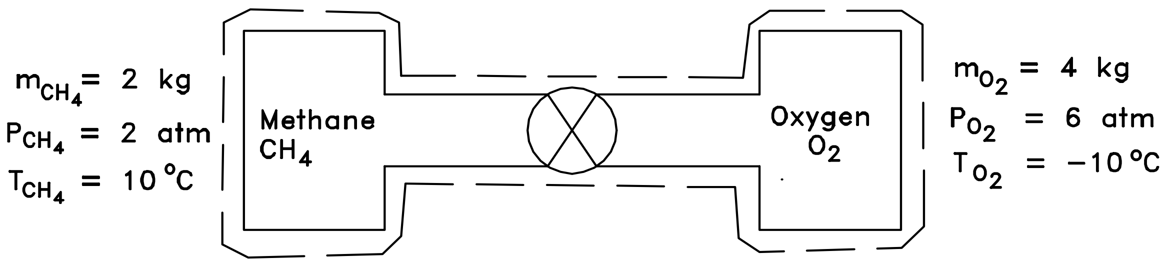
Problem: 10-13

Given: One tank with 2 kg of methane at 2 atm, 10 $^\circ C$.
Another with 4 kg of oxygen at 6 atm, - 10 $^\circ C$.
A valve is opened and the gases mix adiabatically.

Find: Final mixture temperature and pressure.



Energy balance: ($\Delta ke, \Delta pe$, negligible)

$$(m_{CH_4}u_{CH_4})_1 + m_{O_2}u_{O_2})_1 + \cancel{W}^{\,0} + \cancel{Q}^{\,0} = (m_{CH_4}u_{CH_4})_2 + (m_{O_2}u_{O_2})_2$$

Therefore

$$m_{CH_4}(u_1 - u_2)_{CH_4} = m_{O_2}(u_1 - u_2)_{O_2}$$

But $\Delta u = c_v \Delta T$

$$(m_{CH_4}c_{v_{CH_4}})(T_{1_{CH_4}} - T_f) = (m_{O_2}c_{v_{O_2}})(T_f - T_{1_{O_2}})$$

$$T_f = \frac{(mc_vT_1)_{CH_4} + (mc_vT_1)_{O_2}}{(mc_v)_{CH_4} + (mc_v)_{O_2}}$$

From Table B.6b

$$\begin{aligned} c_{v_{CH_4}} &= 1.709\ kJ/kg \cdot K \\ c_{v_{O_2}} &= 0.657\ kJ/kg \cdot K \end{aligned}$$

$$\begin{aligned} T_f &= \frac{(2\ kg)(1.709\ kJ/kg \cdot K)(283.16\ K) + (4\ kg)(0.657\ kJ/kg \cdot K)(263.16\ K)}{(2\ kg)(1.709\ kJ/kg \cdot K) + (4\ kg)(0.657\ kJ/kg \cdot K)} \\ &= \boxed{274.5}\ K \\ &= 1.31\ ^\circ C \end{aligned}$$

$$n = m/\tilde{M}$$

From Table B.6b

$$\begin{aligned}
\tilde{M}_{CH_4} &= 16.04\ kg/kmole \\
\tilde{M}_{O_2} &= 32.00\ kg/kmole
\end{aligned}$$

$$\begin{aligned}
n_{CH_4} &= m_{CH_4}/\tilde{M}_{CH_4} = 2\ kg/16.04\ kg/kmole = 0.1247\ kmole \\
n_{O_2} &= m_{O_2}/\tilde{M}_{O_2} = 4\ kg/32.00\ kg/kmole = 0.125\ kmole \\
V_{CH_4}(initial) &= \frac{NRT_{CH_4}(1)}{P_{CH_4}(1)} = \frac{(0.1247\ kmole)(8314\ J/kmole \cdot K)(283.16\ K)}{(2\ atm)(101,325\ Pa/atm)} \\
&= 1.4485\ m^3 \\
V_{O_2}(initial) &= \frac{NRT_{O_2}(1)}{P_{O_2}(1)} = \frac{(0.125\ kmole)(8314\ J/kmole \cdot K)(263.16\ K)}{(6\ atm)(101,325\ Pa/atm)} \\
&= 0.4499\ m^3 \\
V(final) &= V_{CH_4} + V_{O_2} = 1.4485 + 0.4499 = 1.8984\ m^3
\end{aligned}$$

$$\begin{aligned}
P(final) &= \frac{NRT(final)}{V(final)} \\
&= \frac{(0.2497\ kmole)(8314\ J/kmole \cdot K)(274.47\ K)}{(1.8984\ m^3)} \\
&= 300,136\ Pa \\
&= 0.30\ MPa \\
&= \boxed{2.96\ atm}
\end{aligned}$$